an exact expression for the thermal variation of the emitter base voltage of bi-polar transistors*

by R. J. Widlar

Over the years, a number of interesting and useful properties concerning the highly predictable nature of the emitter-base voltage of bipolar transistors have been discovered. First, it was shown that relationship between the collector current and the emitter-base voltage exactly followed the diode equation over more than eight decades [1], [2]. This has been put directly to use in making logarithmic amplifiers with extremely wide dynamic ranges [3]. Secondly, the theoretically predicted behavior of the emitter-base voltage has made possible dc amplifiers with drifts an order of magnitude lower than can be obtained with conventional techniques [4]. This same theory has been used to produce an ultra-stable, temperature-compensated reference element [5] as well as a low-value current source which does not require large resistance values, making it well suited for integrated circuitry [6], [7]. Lastly, it has been shown that exact temperature compensation of the transconductance can be realized [8]. This has a number of applications including a differential input stage where common mode rejection and gain bandwidth product must be maximized and offset voltage and thermal drift must be minimized, as in a core-memory sense amplifier [9].

This letter will demonstrate another predictable property of the emitter-base voltage in deriving an expression for the emitter-base voltage as a function of temperature in terms of physical constants and the emitter-base voltage at any one temperature. Since a direct attack on this problem quickly produces unwieldly equations, a back-door approach, based on the transconductance-compensation scheme mentioned previously, will be used.

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It is shown in [9] that, assuming very high current gains and low-level injection conditions, the voltage gain of the differential amplifier in Fig. 1 will be constant with temperature for

$$V_B = V_{g0} + (n-1)\frac{kT}{q}. \tag{1}$$

Where V_{g0} is the extrapolated energy gap for the semiconductor material at absolute zero, q is the charge of an electron, n is a constant which depends on how the transistor is made, k is Boltzmann's constant and T is absolute temperture. The transconductance of the differential pair is

$$\frac{\partial Ic}{\partial V_{BE}} = \frac{qI_C}{2kT}, \tag{2}$$

so the gain of the stage is

$$A_V = \frac{qR\,I_C}{2kT}. \tag{3}$$

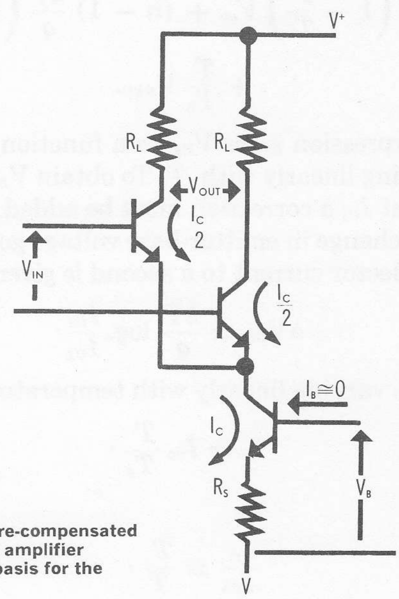

Figure 1
Temperature-compensated differential amplifier used as a basis for the derivation

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In order to have temperature-stable gain, it must be true that

$$I_C \propto T. \tag{4}$$

From Fig. 1,

$$I_C = \frac{V_B - V_{BE}}{R_S} = BT, \tag{5}$$

where B is an arbitrary constant. Hence,

$$V_{BE} = V_B - BTR_S. \tag{6}$$

For $T = T_0$,

$$V_{BEO} = V_B - BT_0R_S, \tag{7}$$

and

$$B = \frac{V_B - V_{BEO}}{T_0R_S}. \tag{8}$$

Substitution into equation (6), this becomes

$$V_{BE} = \left(1 - \frac{T}{T_0}\right)V_B + \frac{T}{T_0}V_{BEO}. \tag{9}$$

Using equation (1) for V_B,

$$V_{BE} = \left(1 - \frac{T}{T_0}\right) V_{g0} + (n-1)\frac{kT}{q}\left(1 - \frac{T}{T_0}\right) + \frac{T}{T_0} V_{BEO}. \tag{10}$$

This expression gives V_{BE} as a function of T for I_C varying linearly with T. To obtain $V_{BE}(T)$ for constant I_C, a correction must be added.

The change in emitter-base voltage going from one collector current to a second is given by

$$\Delta V_{BE} = \frac{kT}{q} \log_e \frac{I_{C2}}{I_{C1}}. \tag{11}$$

With I_C varying linearly with temperature,

$$I_C = I_{C0}\frac{T}{T_0}. \tag{12}$$

Hence,

$$\frac{I_C}{I_{C0}} = \frac{T_0}{T}, \tag{13}$$

and

$$\Delta V_{BE} = \frac{kT}{q} \log_e \frac{T_0}{T}. \tag{14}$$

This is the change in emitter base voltage caused by maintaining a constant collector current. The complete expression for $V_{BE}(T)$ is then

$$V_{BE} = \left(1 - \frac{T}{T_0}\right) V_{g0} + V_{BE0}\frac{T}{T_0} + \frac{kT}{q}\log_e \frac{T_0}{T} + (n-1)\frac{kT}{q}\left(1 - \frac{T}{T_0}\right) \tag{15}$$

The extrapolated energy gap (V_{g0}) for silicon is 1.205 V, k/q has a value of $8.66 \times 10^{-5}\,V/°C$ and the constant n, has a typical value of 1.5 for double-diffused silicon transistors.

To give some appreciation for the magnitude of the terms in (15) a sample calculation can be made for $T_0 = 25°C$. $V_{BEO} = 670$ mV and $T = 125°C$:

$$V_{BE} = -0.403 + 0.894 - 0.010 - 0.006 = 0.476 \text{ V}.$$

This shows that the last two terms of (15) are relatively small, making V_{BE} nearly a linear function of T as is popularly assumed.

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